Math 647 – Applied PDE
Homework 1 – Review of Prerequisites
Due Thursday, January 24 at 1pm

Spring 2019

1. Let $f(x, y) = \sin(xy) - x^3y + xy^4 - 12 + e^x$.
   (a) Compute f_x and f_y.
   (b) Compute[1] f_{xx}, f_{xy}, f_{yx}, and f_{yy}.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \cos(x) + x^2$ and let $u : \mathbb{R}^2 \to \mathbb{R}$ be given by $u(x, y) = x^2y + 2x + y^3$. Use the chain rule compute $\nabla f(u(x, y))$.

3. Find the general solution to the ODE

$$ty' + 2y = e^{t^2}.$$

4. Given any $\alpha \in \mathbb{R}$, solve the initial value problem

$$y' = y^2 \cos(x), \quad y(0) = \alpha.$$

   For what values of α is the solution defined for all time? (*Hint: You may need to treat* $\alpha = 0$ *and* $\alpha \neq 0$ *separately.*)

5. Find the general solution to the differential equation

$$y'' + 2y' + 5y = 0.$$

6. Consider a second order linear homogeneous ODE of the form

$$y'' + P(x)y' + Q(x)y = 0.$$

   where P and Q are defined on some interval in $\mathbb{R}$. Show that the set of all solutions to this ODE forms a vector space. That is, verify each of the following:
   (i) $y = 0$ is a solution.
   (ii) Given any two solutions y_1 and y_2 and constants $\alpha, \beta \in \mathbb{R}$, the function $\alpha y_1 + \beta y_2$ is also a solution of the ODE.

   In fact, you can show (you don't need to do this here, although it might be good to look up) that the vector space of all solutions to the above ODE has dimension 2 and a basis can be found by finding two solutions y_1 and y_2 that have a non-zero Wronskian[2] at some point where the solutions are defined.

[1] Recall that $f_{xy} = (f_x)_y$ and $f_{yx} = (f_y)_x$.
[2] Recall the Wronskian is a determinant that tests for linear independence.