To make above argument mathematically satisfactory, we must...

① Find completion of H w.r.t. $\|\cdot\|_{H_\epsilon}$ (This will be a Sobolev space) so we can apply Riesz Rep.

② Riesz-Rep gives soln. in $\overline{H}^{\|\cdot\|_H}$, not in H. Hence, ~~the~~ soln. may not be smooth! Not clear then how to interpret quantities like Du - leads to theory of weak (or distnl.) derivatives.

③ Continuing, if soln. u from Riesz-Rep is not smooth, then (**) does not hold so may not have

$$f = -\Delta u$$

classically. Thus, need to prove soln. is actually smooth - this is reg. theory (Hard!)

☆ Hallmark of modern analysis —

If you want to guarantee $\exists$ soln. to a given problem, often easier to look in a "bigger" space of fcns. than where you <u>want</u> the soln. to live.

<u>Ex:</u> $\mathbb{Q}$ roots of poly — first search for roots in $\mathbb{R}$!