Math 951 – Advanced PDE II

Spring 2020
11:00am-12:15pm MW, Snow 408

Instructor: Mathew A. Johnson
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Prerequisites: Math 766 and Math 950, or permission of instructor.
Office Hours: Tuesday 10-11, Thursday 11-12, and by appointment.

Course Outline: Our goal is to further develop various aspects of the modern framework of linear and nonlinear partial differential equations. We begin by discussing the theory of Sobolev spaces and relevant some topics in linear operator theory, which provides the functional analytic framework utilized throughout the course. These are then used in the treatment of linear second-order elliptic, parabolic, and hyperbolic equations, such as the kind of potential, diffusion, and wave equations that arise in inhomogeneous media. The emphasis of this part of the class will be on the *solvability* of equations with different initial/boundary conditions, as well as the general qualitative properties of their solutions. We then turn to the study of nonlinear PDE, using several ideas from semigroup theory and nonlinear functional analysis. The course will be valuable to students of differential equations, analysis, probability, and differential geometry.

Tentatively, we plan on covering a selection of the following topics:

- Review of linear operator theory.
- Sobolev spaces: these provide the context in which most modern research on partial differential equations is conducted.
- Second-order linear elliptic equations: Lax-Milgram theorem, existence of weak solutions, regularity, maximum principle, spectral theory of self-adjoint operators (including properties of low eigenvalues and (time permitting) high energy asymptotics).
- Second-order linear parabolic equations: existence of weak solutions, regularity, maximum principle.
- Second-order linear hyperbolic equations: existence of solutions; propagation of waves.
- Nonlinear operator theory techniques in nonlinear PDE: applications of monotone operators, fixed point methods, and bifurcation theory to reaction-diffusion systems, nonlinear elliptic boundary value problems, and nonlinear wave equations.
- Variational methods in nonlinear PDE: criteria for existence of constrained and unconstrained minima of functionals, concentration compactness, and applications to the existence and stability of solutions of nonlinear PDE.
- Linear semigroup theory for evolution equations: infinitesimal generators, Hille-Yosida and Lumer-Phillips theorems, and applications to the existence, well-posedness, and dynamics of linear and nonlinear PDE.

This course seeks to obtain qualitative information on solutions of linear partial differential equations even when explicit solution formulas do not exist. The theories and applications encountered in this course will create a strong foundation for studying nonlinear equations and nonlinear science in general. The latter "nonlinear PDE" material will largely be based on the interests and

abilities of the students.

Grading: Student grades will be based off of several homework assignments and a (possibly group) presentation at the end of the semester.

Required Text: None.

Additional Source Material:

- R. A. Adams, Sobolev Spaces.
- L. C. Evans, Partial Differential Equations.
- R. C. McOwen, Partial Differential Equations: Methods and Applications, second edition
- M. Renardy and R. Rogers, An Introduction to Partial Differential Equations.
- D. Gilbarg and N. Trudinger, Elliptic Partial Differential Equations of Second Order.
- A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations.
- P. D. Lax, Functional Analysis.
- J. Hunter and B. Nachtergaele, Applied Analysis.

Prerequisites: A strong grasp of real analysis (Math 765 and 766) and fluency in multivariable calculus are essential. Some familiarity with basic partial differential equations and measure theory would be useful, but are not strictly required provided you are willing to work hard. Throughout the course we will review (as necessary) basic properties of measure theory and functional analysis, such as Banach and Hilbert spaces and linear operators defined on them.

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Policy on religious observances: Any student in this course who plans to observe a religious holiday that conflicts in any way with the course schedule or requirements should contact the instructor as soon as possible to discuss alternative accommodations.

Policy on Commercial Note Taking: Pursuant to the University of Kansas Policy on Commercial Note-Taking Ventures, commercial note-taking is not permitted in this class. Lecture notes and course materials may be taken for personal use, for the purpose of mastering the course material, and may not be sold to any person or entity in any form. Any student engaged in or contributing to the commercial exchange of notes or course materials will be subject to discipline, including academic misconduct charges, in accordance with University policy. Please note: note-taking provided by a student volunteer for a student with a disability, as a reasonable accommodation under the ADA, is not the same as commercial note-taking and is not covered under this policy.

Policy on Campus Concealed Carry: Individuals who choose to carry concealed handguns **are soley responsible to do so in a safe and secure manner in strict conformity with state and federal laws and KU weapons policy.** Safety measures outlined in the KU weapons policy specify that a concealed handgun:

- Must be under the constant control of the carrier.
- Must be out of view, concealed either on the body of the carrier, or backpack, purse, or bag that remains under the carrier's custody and control.
- Must be in a holster that covers the trigger area and secures any external hammer in an uncocked position.
- Must have the safety on, and have no round in the chamber.